Mixing of liquids with a rotating current density

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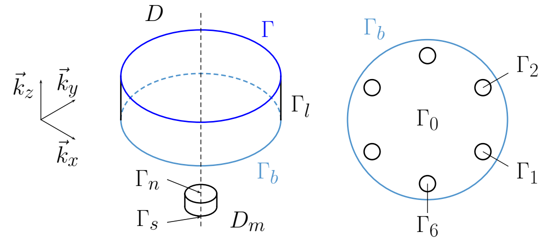
Numerical tools for the study of mixing of small quantities of liquid using a rotating current density and a fixed magnetic field are developed and applied to obtain chaotic advection. A comparison with experiment is done. And the period of the rotating current that maximizes the perimeter of a blob in a dye blob experiment is determined.

Geometry of the rotating current device



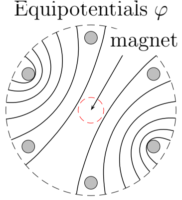
D_m : the magnet ; D : the tank

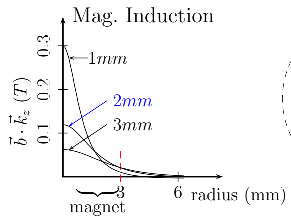
$\Gamma_1, \ldots, \Gamma_6$: 6 electrodes on the bottom of the tank

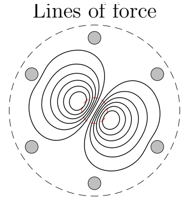
3-phase voltage system on the electrodes

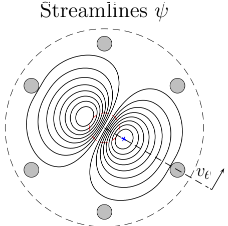
Γ : the free boundary of the electrolyte

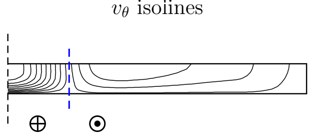
Draws of fields in presence at a frozen time











Equations of fields in presence

Electrical potential

$$\begin{cases} \vec{\nabla}\cdot\sigma\,\vec{\nabla}\varphi = 0 \text{ in } D \\ \varphi = e_n \quad \text{on } \Gamma_n \ : \ n = 1\ldots 6 \\ \partial_n\,\varphi = 0 \quad \text{on } \Gamma + \Gamma_l + \Gamma_0 \end{cases}$$

$$e_n = \Re\{\underline{E}\ \exp^{j\,(\omega\,t + (n-1)\,\pi/6)}\}$$

Magnetical scalar potentiel

$$\phi(\vec{x}) = \frac{\mu_0\,M}{4\pi}\left(\int_{\Gamma_n}\frac{d\vec{x}'^2}{|\vec{x}-\vec{x}'|} - \int_{\Gamma_s}\frac{d\vec{x}'^2}{|\vec{x}-\vec{x}'|}\right)$$

current density : $\vec{j} = -\sigma\,\vec{\nabla}\phi$

Magnetic flux density : $\vec{b} = \vec{\nabla}\Phi$

Lorentz force density : $\vec{f} = \vec{j}\times\vec{b}$

Velocity Field

$$\begin{cases} \mu\vec{\nabla}\times\vec{\nabla}\times\vec{v} + \vec{\nabla}P = \vec{f} - \rho\,g\,\vec{k}_z \\ \vec{\nabla}\cdot\vec{v} = 0 \\ \vec{v} = u\,\vec{k}_x + v\,\vec{k}_y + w\,\vec{k}_z & (Comp.) \\ u = v = w = 0 & \text{on } \Gamma_b + \Gamma_l \\ w = 0\ ;\ \partial_n u = \partial_n v = 0 & \text{on } \Gamma \end{cases}$$

Surface velocity and Stream Function

$\vec{v}_\partial = u|_\Gamma\,\vec{k}_x + v|_\Gamma\,\vec{k}_y$ on Γ

ψ minimizes

$$\int_\Gamma\left((u|_\Gamma - \partial_y\psi)^2 + (v|_\Gamma + \partial_x\psi)^2\right)\,dx\,dy$$

Advection

2D advection equation of a concentration : $\partial_t c_\delta + \vec{\nabla}\cdot(c_\delta\,\vec{v}_\partial) = 0$

The blob B_0 concentration : $c^0_\partial = \{c_0 \text{ in } B_0\ ;\ 0 \text{ in } \Gamma - B_0\}$

If $\vec{\nabla}\cdot\vec{v}_\partial = 0$, $c_\partial = \{c_0 \text{ in } B\ ;\ 0 \text{ in } \Gamma - B\}$ only the boundary ∂B of B has to be computed

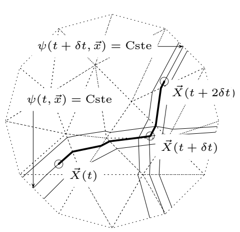
For $X\,\vec{k}_x + Y\,\vec{k}_y \in \partial B$ where $X_0\,\vec{k}_x + Y_0\,\vec{k}_y \in \partial B_0$

$$\begin{cases} \dfrac{dX}{dt} = \partial_y\psi(t,X,Y)\ ; & \dfrac{dY}{dt} = -\partial_x\psi(t,X,Y) \\ X(0) = X_0 & ;\ Y(0) = Y_0 \end{cases}$$

During a time step, the time is frozen and (X, Y) moves along a streamlines, which is piecewise linear.

The points of the polyline ∂B are advected by the EDO with a tactique to increase or decrease the total number of points : if the distance of two neighbors is

- two low, they are merged ;
- two large, an intermediate point is added.



Numerical details

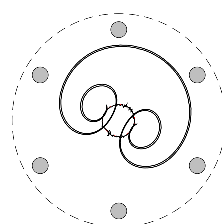
ϕ are computed in 3D with finite element method

The integrales $b_x = \partial_x\Phi, b_y = \partial_y\Phi, \ldots$ are computed on each node of the mesh

u, v, w, P such as $\min_{\vec{v}}\max_{P}\left(\int_D\left(\frac{1}{2}\mu\,(\vec{\nabla}\times\vec{v})^2 - (\vec{f} - \rho\,g\,\vec{k}_z)\cdot\vec{v}\right)\,d\vec{x}^3 - \int_D P\,(\vec{\nabla}\cdot\vec{v})\,d\vec{x}^3\right)$

The Uzawa algorithm is used

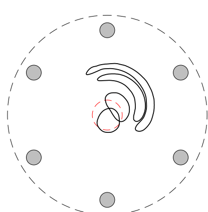
To test the advection algorithm, an initial circular blob is submitted to the stream first down and then upward. The difference between initial and final blobs is 2% root mean square and 10% maximum.



FreeFem++ is used for all computations

Comparison with experiment : fixed direction current

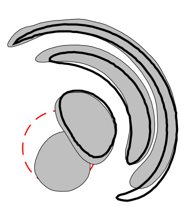
Image processing ⇒ (Imagemagick + Potrace)



R_c, R_a, R_e : radii of tank, magnet and electrodes

H_a, H : heights of liquid and magnet

	R_c	R_a	H_a	H	R_e	$\lvert\underline{E}\rvert$	σ	μ
(mm)	20	3	6	2	1.5	0.7 V	2 S/m	10^{-2} Pa s

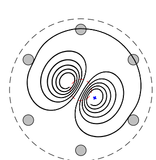


Grey : experiment ; Black lines : computation

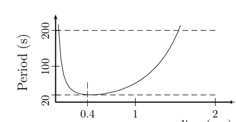
Results : a blob on the magnet, with the perimeter

Blob after 100s

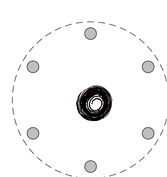
$\omega = 0$



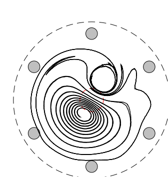
Natural periods vs position



$\omega = 2\pi/5$



$\omega = 2\pi/80$



Two peaks for periods of 5 (within the range of natural periods) and 80 seconds (in the range)

Elongation 50 ×

Perimeter elongation vs $2\pi/\omega$