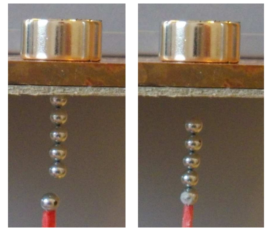
The breakup of a spherical magnetic beads chain suspended along the magnetic axis of a magnet

G. Vinsard, S. Dufour
Université de Lorraine, France

The behavior of magnetic beads submitted to the magnetic field of a cylindrical magnet and pulled is investigated. A finite element model is used to compute the magnetic field and the magnetic flux density, which are used in a recent force formula to obtain results close to those of the experiment.

Harpavat's experiment evidences

Maximum length : 17



Rigidity length : 5

There is a length (in number of beads) of the sustained chain beyond which an extra bead does not cling → *maximum length*

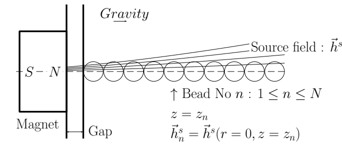
When the chain has more than a certain length a force applied to the below bead breaks the connexion between the chain and this bead and we obtain a chain smaller of a unity → *rigidity length*

Cylindrical magnet : radius 1cm ; height 1cm ; magnetization $0.95\ 10^6$ A/m.

Beads : radius R=2mm ; weight 0.35 g.

Gap between to bead and magnet : 7.6mm.

Mutual Dipole Model = MDM



$$m_n = \gamma_\mu \left(h^s_n + \sum_{p \neq n} \frac{2\ R^3\ m_p}{3\ |z_p - z_n|^3} \right) \ ;\ \gamma_\mu = \frac{3\ (\mu_r - 1)}{\mu_r + 2} \rightarrow N \text{ Eq. } N \text{ Unknowns } m_n$$

$$f_n = \mu_0 \frac{4\ \pi\ R^3}{3} \left(\frac{dh^s_z}{dz}(0, z_n)\ m_n - \sum_{p \neq n} \frac{2\ R^3\ m_n\ m_p}{(z_n - z_p)\ |z_n - z_p|^3} \right) \leftarrow \text{Force on bead No } n$$

- The mean values of the total magnetic field (source + those due to the other beads) is used to determinate the dipolar magnetization in each beads
- The force in each bead is computed with any valid method.

Numerical Model = NM

- D_n the region of the bead No n : $\mu(\vec{x}) = \mu_0\ \mu_r$ for $\vec{x} \in \bigcup_n D_n$; μ_0 otherwise
- Force density on ∂D_n : Bossavit's formula

$$\vec{f}_{\partial D_n} = \frac{1}{2} \left(\mu_0\ [\mu_r]\ \left(\vec{n} \times \left((\vec{h}^s + \vec{h}) \times \vec{n} \right) \right)^2 + \frac{1}{\mu_0} [\nu_r]\ \left((\mu_0\ \vec{h}^s + \vec{b}) \cdot \vec{n} \right)^2 \right)\ \vec{n}$$

- $\vec{h}_s$: source magnetic field ; $\vec{h} = \vec{\nabla}\phi$, $\vec{b} = \vec{\nabla} \times \vec{a}$ magnetic field and magnetic flux density produced by the beads. With azimuthal symmetry : $\phi(r,z)$, $\vec{a} = a(r,z)\ \vec{k}_\theta$

where
$$\begin{cases} \forall\ \vec{b}' = \vec{\nabla} \cdot \left(a'\ \vec{k}_\theta \right) : \int_0^\infty r\ dr \int_{-\infty}^{\infty} dz \left(\frac{\vec{b} \cdot \vec{b}'}{\mu} - \frac{\mu - \mu_0}{\mu} \vec{h}^s \cdot \vec{b}' \right) = 0 \\ \forall\ \vec{h}' = \vec{\nabla}\varphi' \quad : \int_0^\infty r\ dr \int_{-\infty}^{\infty} dz \left(\mu\ \vec{h} \cdot \vec{h}' + (\mu - \mu_0) \vec{h}^s \cdot \vec{h}' \right) = 0 \end{cases}$$

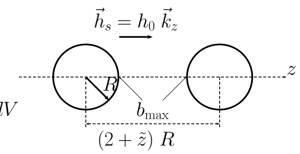
- Force in bead No n : $\vec{f}_n = \int_{\partial D_n} \vec{f}_{\partial D_n}\ dS$
- Lagrange P1 finite element in a truncated region subset of the half-plane $\mathbf{R}^+ \times \mathbf{R}$

Two beads - uniform source field

μ_r	$\tilde{z}$	$\frac{m^*}{h_0}$	$\frac{m^\dagger}{h_0}$	$\frac{f^*_z}{\mu_0\ R^2\ h_0}$ (MDM)	$\frac{f^\dagger_z = f^\dagger_{zb} + f^\dagger_{zh}}{\mu_0\ R^2\ h_0}$ (NM)	$\frac{b_{max}}{\mu_0\ h_0}$ (NM)
0.1	0.5	-1.22	-1.22	0.32	0.29=0.02+0.27	0.09
0.1	0.01	-1.16	-1.17	0.69	0.51=0.48+0.03	0.04
10	0.5	2.49	2.49	1.33	1.55=1.53 +0.02	3.07
10	0.01	2.76	2.95	3.92	14.79=13.01+1.78	14.35
100	0.5	3.32	3.32	2.37	2.89=2.84+0.05	3.98
100	0.01	3.83	4.58	7.51	88.59=87.48+1.11	76.2
∞	0.5	3.44	3.44	2.53	3.10=3.10+0	2.28
∞	0	4	5.69	8.37	2615=2615+0	∞

MDM: $m = \frac{3\,\gamma_\mu\,h_0\,|2\ R + \Delta z|^3}{3\,|2\ R + \Delta z|^3 - 2\,R^3\,\gamma_\mu}$

NM: $m = \frac{3}{4\ \pi\ R^2} \int_{D_n} \left(\frac{\vec{b}}{\mu_0} - \vec{h} \right) \cdot \vec{k}_z\ dV$



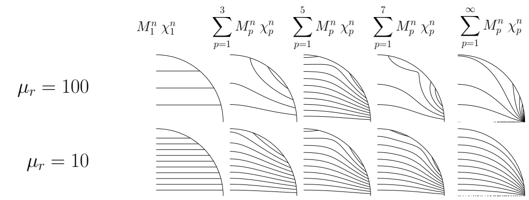
Magnetization in the beads

- In the bead n : $\vec{m}_n = \frac{\vec{b}}{\mu_0} - \vec{h}$ that can be expanded as $\vec{m}_n = \sum_{p=1}^{\infty} M^n_p\ \vec{u}^n_p$ where

$\vec{u}^n_p = \vec{\nabla} \left(\frac{\rho^p}{R^{p-1}}\ P_p(\cos\psi) \right)$ with $\begin{cases} r = \rho\ \sin\psi \\ z = z_n + \rho\ \cos\psi \end{cases}$, P_p : Legendre polynomial ; and

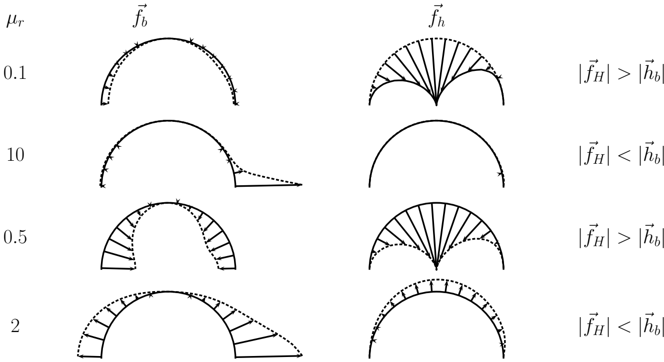
$$M^n_p = \int_{D_n} \vec{m}_n \cdot \vec{u}^n_p\ dV / \int_{D_n} (\vec{u}^n_p)^2\ dV$$

- $\vec{u}^n_p = \vec{\nabla} \times \left(\chi^n_p\ \vec{k}_\theta \right)$: iso-χ^p_n are the lines of magnetization
- With an ∞ number of beads, the magnetization is the same in the beads :

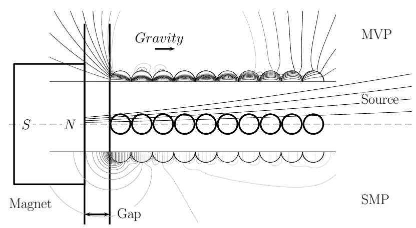


Force density

- Left bead – two beads case :



Characteristic lengths of chains



$\mu_r =$	10	50	100	∞	Exp.
max. length	9	15	16	18	17
rigidity length	3	6	7	9	5

- The values of μ_r that fit best the experimental observations are : 100 for the maximum length and 50 for the rigidity length.
- The difference can be explained if we consider that the magnetic saturation is not taken into account in the model.

Saturation model is needed

- From comparison to experiment ;
- From the inspection of the value of maximum magnetic flux density in the NM ;
- From the asymptotic analysis of a near the contact point of two beads ($z = r = 0$)

$$a \approx A = (r^2 + z^2)/r - r\ \log(r/R)$$

- What is the force density in the non-linear case ?